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13. ABSTRACT (Maximum 200 words)

Research efforts were focused on problems in viscoelasticity, thermoelasticity and thermoviscoelasticity. Results were obtained concerning existence, stability and formation of singularities for classical solutions of various initial-boundary value problems. Some work was also performed on developing models of integral-type for thermoviscoelastic materials.

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Principal Investigator: William J. Hrusa

Research efforts were focussed on problems in thermoelasticity, viscoelasticity, and thermoviscoelasticity. An overview of the research is given below, followed by a list of publications and a description of specific projects, including the work of two graduate students who completed Ph.D. theses under the direction of Hrusa.

I. Thermoelasticity: a) Fourier Heat Conduction: It is well known that smooth motions of nonlinear elastic bodies generally will break down in finite time due to the formation of shock waves. On the other hand, for thermoelastic materials, the conduction of heat provides dissipation that competes with the destabilizing effects of nonlinearity in the elastic response. The work of Coleman & Gurtin (1965) provides a great deal of insight concerning the interplay between dissipation and nonlinearity in one-dimensional nonlinear thermoelasticity. Assuming that the elastic modulus, specific heat, and thermal conductivity are strictly positive, the stress-temperature modulus is nonzero, and that the elastic response is genuinely nonlinear they show that acceleration waves of small initial amplitude decay but waves of large initial amplitude can explode in finite time. In other words, thermal diffusion manages to restrain waves of small amplitude, but nonlinearity in the elastic response is dominant for waves of large amplitude. For initial data that are close to equilibrium (in appropriate Sobolev norms) Slemrod (1981) established global existence and decay of classical solutions to certain initial-boundary value problems in one-dimensional nonlinear thermoelasticity. Slemrod's proof makes crucial use of Poincare's inequality and consequently does not apply to situations in which the interval occupied by the body is unbounded. Subsequently, Zheng & Shen established global existence and decay of classical solutions to the Cauchy problem, in which a nonlinear thermoelastic body occupies the entire real line. The proof of Zheng & Shen combines energy estimates with very complicated Fourier analysis. Because they use both energy methods and Fourier analysis, they make assumptions concerning the initial data that are more restrictive than what is actually needed. In 1988, Hrusa & Tarabek established global

existence and decay of classical solutions to the Cauchy problem by combining the standard energy estimates with some relations that are associated with the second law of thermodynamics. By exploiting the second law of thermodynamics, they were able to simplify the proof and weaken the assumptions made on the initial data. Further, in 1990, Hrusa & Messaoudi showed (for a special class of constitutive relations) that if the initial data are sufficiently far from equilibrium then the solution will develop singularities in finite time. Hrusa & Messaoudi have also established local (in time) existence of smooth solutions for the Dirichlet initial/boundary value problem in three spatial dimensions.

b) Heat Conduction via Cattaneo's Relation: For thermoelastic bodies in which heat conduction is governed by Cattaneo's relation the dissipation of energy is more subtle than for Fourier heat conduction. In 1989, Hrusa & Tarabek established global existence and asymptotic stability of classical solutions to various (nonlinear) initial/boundary value problems with smooth and small initial data.

II. Viscoelasticity: Many of the mathematical results concerning viscoelastic materials require that the kernel (or relaxation function) be "nicely behaved" near zero. However, there are theoretical and experimental indications that certain viscoelastic materials may be well described by equations with kernels that are singular at zero. It turns out that the behavior of solutions to the equations of motion is crucially related to the behavior of the kernel near zero. For nonlinear equations with regular kernels, it is known that smooth solutions exist globally in time if the initial data are sufficiently smooth and small , and that solutions can develop singularities in finite if the initial data are too large. Hrusa (partly in collaboration with Amick) studied a special nonlinear model (in one spatial dimension) with a singular kernel. They worked on establishing global existence of classical solutions for large initial data. An existence theorem has already been established (under assumptions that seem to be stronger than what should really be required). Work is currently underway to try to establish the same result under weaker hypothesis. Another problem studied concerns the global existence of smooth shearing motions of a nonlinear viacoelastic fluid. It is more difficult to establish global existence for a fluid than for a solid because the equilibrium stress modulus is zero for a fluid. In 1990, Brandon & Hrusa established global existence of smooth shearing motions for a nonlinear fluid of K-BKZ type. They assumed that the data are smooth and small and that the kernel is regular. Efforts are currently underway to extend the result to fluids with singular kernels.

III. Thermoviscoelasticity: In 1964, Coleman developed a general theory of thermodynamics of materials with memory. Without assuming a specific form for the constitutive functionals, he obtained necessary and sufficient conditions for compatibility with the second law of thermodynamics. One of his conditions involves a functioinal-differential inequality. In practice, it

is not easy to decide whether or not a given set of constitutive relations is compatible with this relation. Gurtin & Hrusa (1990, 1991) studied nonlinear thermoviscoelastic materials that are described by constitutive relations of single-integral type. They have established simple and direct conditions that are necessary and sufficient in order the constitutive relation are compatible with the second law of thermodynamics. They have also established global exiastence and asymptotic stability of classical solutions to certain initial-value problems with smooth and small initial data. The proof makes novel use of inequalities for matrix-valued kernels of positive type and it exploits several relations that follow from thermodynamics.

Doctoral Students: The following graduate students who have written Ph.D. dissertations under the direction of Hrusa during received partial support from AFOSR 88-0265.

1. Michael Tarabek, On One-Dimensional Nonlinear Thermoelasticity with Second Sound: Existence of Globally Smooth Solutions (1989)

2. Salim Messaoudi, On Local Existence and Formation of Singularities in Nonlinear Thermoelasticity (1989)

The following journal articles were prepared with support from AFOSR 88-0265.

1. W.J. Hrusa & M.A. Tarabek, On smooth solutions of the Cauchy problem in one-dimensional nonlinear thermoelasticity, Q. Appl. Math. **47** (1989), 631-644.

2. W.J. Hrusa & S.A. Messaoudi, On formation of singularities in one-dimensional nonlinear thermoelasticity, Arch. Rational Mech. Anal. **111** (1990), 135-151.

3. M.A. Tarabek, On one-dimensional nonlinear thermoelasticity with Cattaneo's relation. Existence of globally smooth solutions, Q. Appl. Math. (accepted for publication).

4. D. Brandon & W.J. Hrusa, Global existence of smooth shearing motions of a nonlinear viscoelastic fluid, J. Integral Equations Appl. **2** (1990), 333-351.

5. M.E. Gurtin & W.J. Hrusa, On the thermodynamics of viscoelastic materials of single-integral type, Q. Appl. Math **49** (1991), 65-87.

6. M. E. Gurtin & W.J. Hrusa, Global existence in one-dimensional nonlinear viscoelasticity with heat conduction, J. Integral Equations Appl. **2** (1990), 431-460.

A brief description of each publication is given below:

1. We establish global existence and asymptotic stability of classical solutions to the initial-value problem for a one-dimensional nonlinear thermoelastic body (with Fourier heat conduction) that occupies the entire real line, i.e. for the Cauchy problem. The proof, which exploits several identities associated with the second law of thermodynamics is based exclusively on the energy method.

2. We consider the Cauchy problem for a special class of one-dimensional nonlinear thermoelastic materials with Fourier heat conduction. We establish the formation of singularities in finite time for smooth, but large, initial data. Although our constitutive relations are relatively simple, they exhibit the full coupling of thermal and mechanical effects and they are fully compatible with thermodynamics.

3. We establish global existence and asymptotic stability of classical solutions to the equations of motion of a one-dimensional nonlinear thermoelastic body in which the conduction of heat is governed by Cattaneo's relation rather than Fourier's law. The initial data are assumed to be smooth and small. We treat the case when the body is bounded with pinned and insulated ends as well as the case when the body occupies the entire real line.

4. By adapting some of the ideas developed in Brandon's thesis, we establish existence of smooth shearing flows of viscoelastic fluids of the K-BKZ type with smooth and small initial data.

5. We consider a class of nonlinear thermoviscoelastic for which the stress and internal energy depend on the histories of the deformation gradient and temperature through constitutive relations of single-integral type and the heat flux obeys Fourier's Law. We establish direct conditions on the constitutive functions that are necessary and sufficient for compatibility with the second law of thermodynamics.

6. We establish global existence and asymptotic stability of classical solutions to an initial/boundary value problem for a class of one-dimensional nonlinear thermoviscoelastic materials of single-integral type. The data are assumed to be smooth and small. The proof involves energy estimates and inequalities for strongly positive matrix-valued kernels.

Messaoudi's Thesis: Contains the results of (2) above as well as a local existence theorem for the Dirichlet initial/boundary value problem for nonlinear thermoelastic materials (with Fourier heat conduction) in three spatial dimensions.

Tarabek's Thesis: Contains the results of (3) above and an expanded discussion of the relevant physical background for the problem.